$B_s \to \tau^+ \tau^-$ decay in the general two Higgs doublet model

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Abstract

We study the exclusive decay $B_s \to \tau^+ \tau^-$ in the general two Higgs doublet model. We analyse the dependencies of the branching ratio on the model parameters, including the leading order QCD corrections. We found that there is an enhancement in the branching ratio, especially for $r_{tb} = \frac{\bar{\xi}^U_{N,tt}}{\bar{\xi}^D_{N,bb}} > 1$ case. Further, the neutral Higgs effects are detectable for large values of the parameter $\bar{\xi}^D_{N,\tau\tau}$.

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1 Introduction

The study of rare B-decays is one of the most important research areas in particle physics and there is an experimental effort for studying them at various centers such as SLAC (BaBar), KEK (BELLE), B-Factories, DESY (HERA-B). In the Standard model (SM) they are induced by flavor changing neutral currents (FCNC) at loop level and therefore they are sensitive to the fundamental parameters, like Cabbibo-Kobayashi-Maskawa (CKM) matrix elements, leptonic decay constants, etc. These decays also provide a sensitive test to the new physics beyond the SM, such as two Higgs Doublet model (2HDM), Minimal Supersymmetric extension of the SM (MSSM) [1], etc.

Among the rare B decays, $B_s \rightarrow l^+ l^-$ process, induced by the inclusive $b \rightarrow s l^+ l^-$ decay, is attractive since the only non-perturbative quantity in the theoretical calculation is the decay constant of B_s which is reliably known. From the experimental point of view, the measurement of the hadronic decay is easier compared to its inclusive channel.

The measurement of upper limit of $B_s \rightarrow \mu^+ \mu^-$ [2]

$$BR(B_s \rightarrow \mu^+ \mu^-) \leq 2.6\, 10^{-6} \; , \tag{1}$$

stimulated the study of $B_s \rightarrow l^+ l^-$ decays. In the literature, this process ([3]-[9]) and its inclusive one $B_s \rightarrow X_s l^+ l^-$ ([10]-[13]) have been investigated extensively in the SM, 2HDM and supersymmetric model (SUSY). When $l = e, \mu$, the neutral Higgs boson (NHB) effects are safely neglected in the 2HDM because they enter in the expressions with the factor $m_{e(\mu)}/m_W$, $m_{H^\pm}$. However, for $l = \tau$, this factor is not negligible and NHB effects can give important contribution. In [13], $B \rightarrow X_s \tau^+ \tau^-$ process was studied in the 2HDM and it was shown that NHB effects are sizable for large values of $tan\beta$. Therefore the main observation of these calculations is the enhancement of the branching ratio (BR) of these decays for large $tan\beta$ values in the 2HDM and minimal supersymmetric model (MSSM), especially for $l = \tau$ lepton case. In a recent work [14], the inclusive $b \rightarrow s \tau^+ \tau^-$ process has been studied in the general 2HDM with real Yukawa couplings and it was found that the BR has been enhanced for large values of the parameters $\bar{\xi}^D_{N,\tau\tau}$ and $\bar{\xi}^D_{N,bb}$. In this work, we study the $B_s \rightarrow \tau^+ \tau^-$ decay in the general 2HDM, so-called model III . Our calculations are based on the results of the work [14] for the inclusive $b \rightarrow s \tau^+ \tau^-$ decay. Here we include NHB effects and make the full calculation using the on-shell renormalization prescription. The investigation of the dependencies of the BR on the model parameters, namely $\bar{\xi}^D_{N,bb}$ and $\bar{\xi}^D_{N,\tau\tau}$, shows that a large enhancement in the BR is possible.

The paper is organized as follows: In Section 2, we present the leading order (LO) QCD

corrected effective Hamiltonian and the corresponding matrix element for the exclusive $B_s \to \tau^+\tau^-$ decay in the framework of the model III. Section 3 is devoted to the analysis of the dependencies of the BR on the the Yukawa couplings $\bar{\xi}^D_{N,bb}$, $\bar{\xi}^D_{N,\tau\tau}$ and to the discussion of our results. In Appendices, we present the operators appearing in the effective Hamiltonian and their Wilson coefficients.

2 The $B_s \to \tau^+\tau^-$ decay in the framework of the model III

The general Yukawa interaction in the model III is

$$\mathcal{L}_Y = \eta^U_{ij}\bar{Q}_{iL}\tilde{\phi}_1 U_{jR} + \eta^D_{ij}\bar{Q}_{iL}\phi_1 D_{jR} + \xi^U_{ij}\bar{Q}_{iL}\tilde{\phi}_2 U_{jR} + \xi^D_{ij}\bar{Q}_{iL}\phi_2 D_{jR} + h.c. \ , \tag{2}$$

where L and R denote chiral projections $L(R) = 1/2(1 \mp \gamma_5)$ and ϕ_i for $i = 1, 2$, are two scalar doublets. Here $\eta^{U,D}_{ij}$, $\xi^{U,D}_{ij}$ are the Yukawa matrices and, in general, they have complex entries. The choice of scalar Higgs doublets

$$\begin{aligned}\phi_1 &= \frac{1}{\sqrt{2}}\left[\left(\begin{array}{c} 0 \\ v + H^0 \end{array}\right) + \left(\begin{array}{c} \sqrt{2}\chi^+ \\ i\chi^0 \end{array}\right)\right] \ , \\ \phi_2 &= \frac{1}{\sqrt{2}}\left(\begin{array}{c} \sqrt{2}H^+ \\ H^1 + iH^2 \end{array}\right) \ ,\end{aligned} \tag{3}$$

with the vacuum expectation values,

$$< \phi_1 >= \frac{1}{\sqrt{2}}\left(\begin{array}{c} 0 \\ v \end{array}\right) \ ; < \phi_2 >= 0 \ , \tag{4}$$

and the gauge and CP invariant Higgs potential which spontaneously breaks $SU(2) \times U(1)$ down to $U(1)$:

$$\begin{aligned} V(\phi_1, \phi_2) &= c_1(\phi_1^+\phi_1 - v^2/2)^2 + c_2(\phi_2^+\phi_2)^2 \\ &+ \ + c_3[(\phi_1^+\phi_1 - v^2/2) + \phi_2^+\phi_2]^2 + c_4[(\phi_1^+\phi_1)(\phi_2^+\phi_2) - (\phi_1^+\phi_2)(\phi_2^+\phi_1)] \\ &+ \ c_5[Re(\phi_1^+\phi_2)]^2 + c_6[Im(\phi_1^+\phi_2)]^2 + c_7 \, , \end{aligned} \tag{5}$$

where constants c_i, $i = 1, ..., 7$, permits us to carry the SM particles in the first doublet and the information about the new physics by the second one. The Yukawa interaction

$$\mathcal{L}_{Y,FC} = \xi^U_{ij}\bar{Q}_{iL}\tilde{\phi}_2 U_{jR} + \xi^D_{ij}\bar{Q}_{iL}\phi_2 D_{jR} + h.c. \ . \tag{6}$$

describes the Flavor Changing (FC) one beyond the SM. Here, the couplings $\xi^{U,D}$ for the charged FC interactions are

$$
\begin{aligned}
\xi^{U}_{ch} &= \xi^{U}_{N}\, V_{CKM} \,, \\
\xi^{D}_{ch} &= V_{CKM}\, \xi^{D}_{N} \,,
\end{aligned} \tag{7}
$$

and

$$
\xi^{U,D}_{N} = (V_L^{U,D})^{-1} \xi^{U,D}\, V_R^{U,D} \,, \tag{8}
$$

where the index "N" in $\xi^{U,D}_{N}$ denotes the word "neutral". Notice that H_1 and H_2 are the mass eigenstates h^0 and A^0 respectively, since no mixing occurs between two CP-even neutral bosons H^0 and h^0 in the tree level, for our choice.

The exclusive $B_s \to \tau^+\tau^-$ process is induced by the inclusive $b \to s\tau^+\tau^-$ decay. Therefore we start with the effective Hamiltionian of $b \to s\tau^+\tau^-$ decay

$$
\begin{aligned}
\mathcal{H}_{eff} &= \frac{\alpha G_F}{\sqrt{2}\,\pi} V_{tb} V^*_{ts} \Big\{ C_9^{eff} (\bar{s}\gamma_\mu P_L b)\, \bar{\tau}\gamma_\mu \tau + C_{10} (\bar{s}\gamma_\mu P_L b)\, \bar{\tau}\gamma_\mu\gamma_5 \tau \\
&- 2 C_7 \frac{m_b}{p^2} (\bar{s} i \sigma_{\mu\nu} p_\nu P_R b) \bar{\tau}\gamma_\mu \tau + C_{Q_1} (\bar{s} P_R b) \bar{\tau}\tau + C_{Q_2} (\bar{s} P_R b) \bar{\tau}\gamma_5 \tau \Big\} \,,
\end{aligned} \tag{9}
$$

with $p = p_1 + p_2$, the sum of the momenta of τ^+ and τ^-. Note that, C_7, C_9 and C_{10}, are the Wilson coefficients normalized at the scale μ and given in Appendix B. The additional Wilson coefficients C_{Q_1} and C_{Q_2} are due to the NHB exchange diagrams (see Appendix B). In calculating the $\mathcal{H}_{eff}$, one first integrates out the heavy degrees of freedom, namely t quark, $W^\pm$, $H^\pm$, H_0, H_1 and H_2 bosons in the present case and then obtain the effective theory. Here $H^\pm$ denote charged, and H^0, H_1 ,H_2 denote neutral Higgs bosons. Note that H_1 and H_2 are the same as the mass eigenstates h^0 and A^0 in the model III respectively. At this stage the QCD corrections are added through matching the full theory with the effective low energy one at the high scale $\mu = m_W$ and evaluating the Wilson coefficients from m_W down to the lower scale $\mu \sim O(m_b)$.

In the model III, the neutral Higgs particles bring additional contributions (see eq.(19)) since the mass of τ lepton or related Yukawa coupling $\bar{\xi}^D_{N,\tau\tau}$ enter into the expressions (see [14]). Finally the neutral Higgs boson (NHB) contributions are calculated using the on-shell renormalization scheme to overcome the logarithmic divergences. Using the renormalization condition

$$
\Gamma^{Ren}_{neutr}(p^2) = \Gamma^0_{neutr}(p^2) + \Gamma^C_{neutr} = 0, \tag{10}
$$

the counter term Γ^{C}_{neutr} and then the renormalized vertex function $\Gamma^{Ren}_{neutr}(p^2)$ is obtained. Here the phrase *neutr* denotes the neutral Higgs bosons H^0, h^0 and A^0 and p is the momentum transfer.

For the exclusive decay $B_s \to \tau^+\tau^-$, $\mathcal{H}_{eff}$ is to be taken between vacuum and $|B^0_s>$ state as $<0|\mathcal{H}_{eff}|B^0_s>$ and this matrix element can be expressed in terms of the B^0_s decay constant f_{B_s} using

$$\begin{aligned}
<0|\bar{s}\gamma_\mu\gamma_5 b|B^0_s> &= -if_{B_s}p_\mu \ , \\
<0|\bar{s}\gamma_5 b|B^0_s> &= if_{B_s}\frac{m^2_{B_s}}{m_b+m_s} \ , \\
<0|\bar{s}\sigma_{\mu\nu}P_R b|B^0_s> &= 0. \qquad (11)
\end{aligned}$$

Since $p = p_1 + p_2$, the C_9^{eff} term in eq.(9) gives zero on contraction with the lepton bilinear, C_7 gives zero by eq.(11) and the C_{10} term gets a factor of $2m_\tau$ while the remaining C_{Q_1} and C_{Q_2} terms get m_{B_s}, when taking $m_{B_s} \approx m_b + m_s$. Thus the effective Hamiltonian eq. (9) results in the following decay amplitude for $B_s \to \tau^+\tau^-$

$$A = \frac{G_F\alpha}{2\sqrt{2}\pi} m_{B_s} f_{B_s} V_{tb}V^*_{ts}\left[C_{Q_1}\bar{\tau}\tau + (C_{Q_2} - 2\frac{m_\tau}{m_{B_s}}C_{10})\bar{\tau}\gamma_5\tau\right]. \qquad (12)$$

To calculate the branching ratio we find the square of this amplitude, then sum over the spins of the τ leptons and finally integrate over the phase space. This straightforward calculation gives for the branching ratio of $B_s \to \tau^+\tau^-$

$$BR = \frac{G_F^2\alpha^2}{64\pi^3} m^3_{B_s}\tau_{B_s} f^2_{B_s}|V_{tb}V^*_{ts}|^2\sqrt{1-4\frac{m^2_\tau}{m^2_{B_s}}}\left[(1-4\frac{m^2_\tau}{m^2_{B_s}})|C_{Q_1}|^2 + |C_{Q_2} - 2\frac{m_\tau}{m_{B_s}}C_{10}|^2\right]. \qquad (13)$$

3 Discussion

In the multi-Higgs doublet models, there are many free parameters, such as masses of charged and neutral Higgs bosons and the Yukawa couplings. In the present work we study our process in the general 2HDM, so called model III. The Yukawa couplings, which are entries of Yukawa matrices can be restricted using the experimental measurements. In our calculations, we neglect all Yukawa couplings except $\bar{\xi}^U_{N,tt}$, $\bar{\xi}^D_{N,bb}$, $\bar{\xi}^D_{N,\tau\tau}$ by respecting the CLEO measurement [15],

$$BR(B \to X_s\gamma) = (3.15 \pm 0.35 \pm 0.32)\, 10^{-4} \ . \qquad (14)$$

This section is devoted to $\frac{\bar{\xi}^D_{N,bb}}{m_b}$ and $\bar{\xi}^D_{N,\tau\tau}$ dependencies of BR for the exclusive decay $B_s \to \tau^+\tau^-$, restricting $|C_7^{eff}|$ in the region $0.257 \leq |C_7^{eff}| \leq 0.439$ due to the CLEO measurement,

eq.(14) (see [16] for details). In our numerical calculations, we take the charged Higgs mass $m_{H^\pm} = 400\, GeV$ and the scale $\mu = m_b$. Further, we use the redefinition

$$\xi^{U,D} = \sqrt{\frac{4G_F}{\sqrt{2}}}\bar{\xi}^{U,D} ,$$

and the input values given in Table (1).

Parameter	Value
m_τ	1.78 (GeV)
m_c	1.4 (GeV)
m_b	4.8 (GeV)
m_{H^0}	150 (GeV)
m_{h^0}	80 (GeV)
m_{A^0}	80 (GeV)
α_{em}^{-1}	129
λ_t	0.04
m_t	175 (GeV)
m_W	80.26 (GeV)
m_Z	91.19 (GeV)
Λ_{QCD}	0.225 (GeV)
$\alpha_s(m_Z)$	0.117
$sin\theta_W$	0.2325

Table 1: The values of the input parameters used in the numerical calculations.

Fig. 1 shows $\frac{\bar{\xi}^D_{N,bb}}{m_b}$ dependence of the BR of the decay under consideration for $\bar{\xi}^D_{N,\tau\tau} = 200\, GeV$ and the ratio $|r_{tb}| = |\frac{\bar{\xi}^D_{N,bb}}{\bar{\xi}^U_{N,tt}}| < 1$. The BR is restricted to the region bounded by solid lines for $C_7^{eff} > 0$ or to the small dashed line for $C_7^{eff} < 0$. This quantity is sensitive to $\frac{\bar{\xi}^D_{N,bb}}{m_b}$ and it increases by an amount $\% 60$ in the interval $20 \leq \frac{\bar{\xi}^D_{N,bb}}{m_b} \leq 80$. Besides, the enhancement compared to the SM case is predicted as beeing $\%80$. The $\frac{\bar{\xi}^D_{N,bb}}{m_b}$ dependence of the BR for $r_{tb} > 1$ is presented in Fig. 2. For this case, the BR increases considerably even for small values of $\bar{\xi}^D_{N,\tau\tau}$, which is taken $20\, GeV$, in this calculation. The BR enhances with increasing $\bar{\xi}^D_{N,\tau\tau}$, especially for $C_7^{eff} < 0$ case. This figure shows that the BR is strongly sensitive to the parameter $\bar{\xi}^D_{N,bb}$ for $r_{tb} > 1$ and it may get the values four (two) times larger compared to the ones in the SM for $C_7^{eff} < 0$ ($C_7^{eff} > 0$) even at $\bar{\xi}^D_{N,bb} = 2\, m_b$.

Figures (3-4) represent the dependencies of the BR on the parameter $\bar{\xi}^D_{N,\tau\tau}$ for $|r_{tb}| < 1$ and $r_{tb} > 1$ respectively. In $r_{tb} < 1$ case, the BR increases almost 1.5 times compared to the one in the SM for large values of $\bar{\xi}^D_{N,\tau\tau}$, $\bar{\xi}^D_{N,\tau\tau} = 500\, GeV$ (Fig. 3). However, for $r_{tb} > 1$, this enhancement is quite high as shown in Fig. 4. Even for small values of $\bar{\xi}^D_{N,bb}$ and $\bar{\xi}^D_{N,\tau\tau}$ there

is a possible increase nearly (more than) one order of magnitude compared to the SM case for $C_7^{eff} > 0$ ($C_7^{eff} < 0$).

Now we would like to summarize our results:

- There is a possible enhancement in the BR at the order of magnitude $\%\,150$ for $|r_{tb}| < 1$ in the model III compared to the one in the SM for large values of the model III parameters, $\bar{\xi}^{D}_{N,bb} = 80\, m_b$ and $\bar{\xi}^{D}_{N,\tau\tau} = 500\, GeV$. The BR is not so much sensitive to the model parameters given above. Further, the NHB effects become sizable with increasing values of $\bar{\xi}^{D}_{N,\tau\tau}$.

- For $r_{tb} > 1$, there is a considerable enhancement at the one order of magnitude compared to the SM, even for the small values of $\bar{\xi}^{D}_{N,bb}$ and $\bar{\xi}^{D}_{N,\tau\tau}$. In this case, the BR is larger and more sensitive the model parameters for $C_7^{eff} < 0$ than the ones for $C_7^{eff} > 0$. Note that the enhancement for the increasing values of the $\bar{\xi}^{D}_{N,\tau\tau}$ is due to the dependence of the BR on the NHB effects.

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Appendix

A The operator basis

The operator basis in the 2HDM (model III) for our process is [13, 17, 18]

$$
\begin{aligned}
O_1 &= (\bar{s}_{L\alpha}\gamma_\mu c_{L\beta})(\bar{c}_{L\beta}\gamma^\mu b_{L\alpha}), \\
O_2 &= (\bar{s}_{L\alpha}\gamma_\mu c_{L\alpha})(\bar{c}_{L\beta}\gamma^\mu b_{L\beta}), \\
O_3 &= (\bar{s}_{L\alpha}\gamma_\mu b_{L\alpha}) \sum_{q=u,d,s,c,b} (\bar{q}_{L\beta}\gamma^\mu q_{L\beta}), \\
O_4 &= (\bar{s}_{L\alpha}\gamma_\mu b_{L\beta}) \sum_{q=u,d,s,c,b} (\bar{q}_{L\beta}\gamma^\mu q_{L\alpha}), \\
O_5 &= (\bar{s}_{L\alpha}\gamma_\mu b_{L\alpha}) \sum_{q=u,d,s,c,b} (\bar{q}_{R\beta}\gamma^\mu q_{R\beta}), \\
O_6 &= (\bar{s}_{L\alpha}\gamma_\mu b_{L\beta}) \sum_{q=u,d,s,c,b} (\bar{q}_{R\beta}\gamma^\mu q_{R\alpha}), \\
O_7 &= \frac{e}{16\pi^2}\bar{s}_\alpha\sigma_{\mu\nu}(m_b R + m_s L) b_\alpha \mathcal{F}^{\mu\nu}, \\
O_8 &= \frac{g}{16\pi^2}\bar{s}_\alpha T^a_{\alpha\beta}\sigma_{\mu\nu}(m_b R + m_s L) b_\beta \mathcal{G}^{a\mu\nu} \ , \\
O_9 &= \frac{e}{16\pi^2}(\bar{s}_{L\alpha}\gamma_\mu b_{L\alpha})(\bar{\tau}\gamma^\mu\tau) \ , \\
O_{10} &= \frac{e}{16\pi^2}(\bar{s}_{L\alpha}\gamma_\mu b_{L\alpha})(\bar{\tau}\gamma^\mu\gamma_5\tau) \ , \\
Q_1 &= \frac{e^2}{16\pi^2}(\bar{s}^\alpha_L\, b^\alpha_R)\,(\bar{\tau}\tau) \\
Q_2 &= \frac{e^2}{16\pi^2}(\bar{s}^\alpha_L\, b^\alpha_R)\,(\bar{\tau}\gamma_5\tau) \\
Q_3 &= \frac{g^2}{16\pi^2}(\bar{s}^\alpha_L\, b^\alpha_R) \sum_{q=u,d,s,c,b} (\bar{q}^\beta_L\, q^\beta_R) \\
Q_4 &= \frac{g^2}{16\pi^2}(\bar{s}^\alpha_L\, b^\alpha_R) \sum_{q=u,d,s,c,b} (\bar{q}^\beta_R\, q^\beta_L) \\
Q_5 &= \frac{g^2}{16\pi^2}(\bar{s}^\alpha_L\, b^\beta_R) \sum_{q=u,d,s,c,b} (\bar{q}^\beta_L\, q^\alpha_R) \\
Q_6 &= \frac{g^2}{16\pi^2}(\bar{s}^\alpha_L\, b^\beta_R) \sum_{q=u,d,s,c,b} (\bar{q}^\beta_R\, q^\alpha_L) \\
Q_7 &= \frac{g^2}{16\pi^2}(\bar{s}^\alpha_L\, \sigma^{\mu\nu}\, b^\alpha_R) \sum_{q=u,d,s,c,b} (\bar{q}^\beta_L\, \sigma_{\mu\nu} q^\beta_R) \\
Q_8 &= \frac{g^2}{16\pi^2}(\bar{s}^\alpha_L\, \sigma^{\mu\nu}\, b^\alpha_R) \sum_{q=u,d,s,c,b} (\bar{q}^\beta_R\, \sigma_{\mu\nu} q^\beta_L)
\end{aligned}
$$

$$
\begin{aligned}
Q_9 &= \frac{g^2}{16\pi^2}(\bar{s}_L^{\alpha}\,\sigma^{\mu\nu}\,b_R^{\beta})\sum_{q=u,d,s,c,b}(\bar{q}_L^{\beta}\,\sigma_{\mu\nu}q_R^{\alpha}) \\
Q_{10} &= \frac{g^2}{16\pi^2}(\bar{s}_L^{\alpha}\,\sigma^{\mu\nu}\,b_R^{\beta})\sum_{q=u,d,s,c,b}(\bar{q}_R^{\beta}\,\sigma_{\mu\nu}q_L^{\alpha})
\end{aligned}
\tag{15}
$$

where α and β are $SU(3)$ colour indices and $\mathcal{F}^{\mu\nu}$ and $\mathcal{G}^{\mu\nu}$ are the field strength tensors of the electromagnetic and strong interactions, respectively. Note that there are also flipped chirality partners of these operators, which can be obtained by interchanging L and R in the basis given above in the model III. However, we do not present them here since corresponding Wilson coefficients are negligible.

B The Initial values of the Wilson coefficients.

For the sake of completeness we also give the initial values of the Wilson coefficients for the relevant process. In the SM they are [17]

$$
\begin{aligned}
C^{SM}_{1,3,\ldots 6,11,12}(m_W) &= 0\ , \\
C^{SM}_{2}(m_W) &= 1\ , \\
C^{SM}_{7}(m_W) &= \frac{3x_t^3-2x_t^2}{4(x_t-1)^4}\ln x_t+\frac{-8x_t^3-5x_t^2+7x_t}{24(x_t-1)^3}\ , \\
C^{SM}_{8}(m_W) &= -\frac{3x_t^2}{4(x_t-1)^4}\ln x_t+\frac{-x_t^3+5x_t^2+2x_t}{8(x_t-1)^3}\ , \\
C^{SM}_{9}(m_W) &= -\frac{1}{sin^2\theta_W}B(x_t)+\frac{1-4\sin^2\theta_W}{\sin^2\theta_W}C(x_t)-D(x_t)+\frac{4}{9},\ , \\
C^{SM}_{10}(m_W) &= \frac{1}{\sin^2\theta_W}(B(x_t)-C(x_t))\ , \\
C^{SM}_{Q_i}(m_W) &= 0 \quad i=1,..,10\ .
\end{aligned}
\tag{16}
$$

The initial values for the additional part due to charged Higgs bosons are

$$
\begin{aligned}
C^{H}_{1,\ldots 6}(m_W) &= 0\,, \\
C^{H}_{7}(m_W) &= Y^2\,F_1(y_t)\,+\,XY\,F_2(y_t)\ , \\
C^{H}_{8}(m_W) &= Y^2\,G_1(y_t)\,+\,XY\,G_2(y_t)\ , \\
C^{H}_{9}(m_W) &= Y^2\,H_1(y_t)\ , \\
C^{H}_{10}(m_W) &= Y^2\,L_1(y_t)\ ,
\end{aligned}
\tag{17}
$$

where

$$
X = \frac{1}{m_b}\quad\left(\bar{\xi}^D_{N,bb}+\bar{\xi}^D_{N,sb}\frac{V_{ts}}{V_{tb}}\right)\quad ,
$$

$$Y = \frac{1}{m_t} \left(\bar{\xi}^U_{N,tt} + \bar{\xi}^U_{N,tc} \frac{V^*_{cs}}{V^*_{ts}} \right) \quad , \tag{18}$$

and due to the neutral Higgs bosons are [14]

$$C^{A^0}_{Q_2}((\bar{\xi}^U_{N,tt})^3) = \frac{\bar{\xi}^D_{N,\tau\tau}(\bar{\xi}^U_{N,tt})^3 m_b y_t}{32\pi^2 m^2_{A^0} m_t \Theta_1(z_A)(y_t-1)^2}((y_t-1)(-\Theta_1(z_A)+(y_t-1)z_A)+\Theta_1(z_A)\ln y_t),$$

$$C^{A^0}_{Q_2}((\bar{\xi}^U_{N,tt})^2) = \frac{\bar{\xi}^D_{N,\tau\tau}(\bar{\xi}^U_{N,tt})^2 \bar{\xi}^D_{N,bb}}{32\pi^2 m^2_{A^0}} \left(\frac{(1-2y_t)\ln y_t}{y_t-1} + 2(1+\ln\left[\frac{\Theta_1(z_A)}{z_A}\right]) - \frac{y_t(xy+z_A)}{\Theta_1(z_A)} \right),$$

$$\begin{aligned}
C^{A^0}_{Q_2}(\bar{\xi}^U_{N,tt}) &= \frac{g^2 \bar{\xi}^D_{N,\tau\tau}\bar{\xi}^U_{N,tt} m_b x_t}{128\pi^2 m_t} \Bigg(2z_A \frac{-x(\frac{y}{\Theta_2(z_A)}+\frac{y_t}{\Theta_3(z_A)}) + \frac{y_t(x-1)}{-\Theta_3(z_A)+(x-y)(x_t-y_t)z_A}}{m^2_W} \\
&+ \Bigg(\frac{1}{m^2_{A^0}} (-\frac{4z_A}{\Theta_2(z_A)} + \frac{2(x(x_t+y_t)-2y_t)z_A}{-\Theta_3(z_A)} - \frac{2(x_t(x-1)+(x+1)y_t)z_A}{\Theta_3(z_A)+(x-y)(x_t-y_t)z_A} \\
&+ \frac{(y_t-x_t)z_A + x_t y_t(2z_A-1)}{(x_t-1)(y_t-1)z_A} - \frac{(4x_t^3 - 7y_t - 4x_t^2(2+y_t) + x_t(5+8y_t+\frac{y_t}{z_A}))\ln x_t}{(x_t-1)^2(x_t-y_t)} \\
&+ \frac{(x_t(\frac{y_t}{z_A}-1)-y_t)\ln y_t}{(x_t-y_t)(y_t-1)^2} + 4\ln\left[\frac{\Theta_2(z_A)}{z_A}\right]\Bigg)\Bigg),
\end{aligned}$$

$$\begin{aligned}
C^{A^0}_{Q_2}(\bar{\xi}^D_{N,bb}) = -\frac{g^2 \bar{\xi}^D_{N,\tau\tau}\bar{\xi}^D_{N,bb}}{64\pi^2 m^2_{A^0}} \Bigg(& \frac{2\Theta_3(z_A) - x_t((x-2)yy_t - x_t(y_t-z_A))}{\Theta_3(z_A)} + 2\ln\left[\frac{\Theta_3(z_A)}{z_A}\right] \\
& -\frac{(y_t - 2x_t(y_t+1) + x_t^2(\frac{y_t}{z_A}+1)\ln x_t}{(x_t-1)(x_t-y_t)} + \frac{(x_t(1-2y_t)+2y_t(y_t-1)+x_t^2(\frac{y_t}{z_A}-1))\ln y_t}{(y_t-1)(x_t-y_t)}\Bigg),
\end{aligned}$$

$$\begin{aligned}
C^{H^0}_{Q_1}((\bar{\xi}^U_{N,tt})^2) = -\frac{g^2(\bar{\xi}^U_{N,tt})^2 m_b m_\tau y_t}{256\pi^2 m^2_{H^0} m^2_W x_t} \Bigg(& -\frac{\frac{4xz_H}{\Theta_4(z_H)} + \frac{-1+4y_t+y_t^2(2\ln y_t - 3)}{(y_t-1)^3}}{\cos^2\theta_W} + \\
& 2\Big(\frac{2z_H(-\Theta_4(z_H)(1-2x)x_t + 2\Theta_1(z_H)x)}{\Theta_1(z_H)\Theta_4(z_H)} - \frac{(y_t-1)(1+x_t+y_t(x_t-3)) + 2y_t(y_t-x_t)\ln y_t}{(y_t-1)^3}\Big)\Bigg),
\end{aligned}$$

$$\begin{aligned}
C^{H^0}_{Q_1}(\bar{\xi}^U_{N,tt}) = \frac{g^2 \bar{\xi}^U_{N,tt}\bar{\xi}^D_{N,bb} m_\tau}{64\pi^2 m^2_{H^0} m_t} \Bigg(& \frac{y_t(2-x_t)-x_t}{y_t-1} + 2x_t\ln\left[\frac{\Theta_1(z_H)}{z_H}\right] - \frac{(x_t(1-5y_t)+2y_t^2(1+x_t))\ln y_t}{(y_t-1)^2} \\
& -\frac{z_H}{\Theta_1(z_H)\Theta_4(z_H)}\Big(-x^2 x_t\Theta_5(y_t-1) + x(x_t(-\Theta_4(z_H)+\Theta_5(1+x-y))(y_t-1) \\
& + 2y_t(-\Theta_5+2yy_t)) + (-x_t\Theta_6(y_t-1) - 2(1+y(y_t-1))y_t)z_H\Big) \\
& + \frac{y_t((y_t-1)(-\Theta_4(z_H)+z_H(1-y_t)) + \Theta_4(z_H)y_t\ln y_t)}{\cos^2\theta_W\Theta_4(z_H)(y_t-1)^2}\Bigg),
\end{aligned}$$

$$\begin{aligned}
C^{H^0}_{Q_1}(g^4) = -\frac{g^4 m_b m_\tau}{512\pi^2 m^2_{H^0} m^2_W} \Bigg(& \frac{\frac{4(x-1)}{\Theta_7} + \frac{x_t(x_t(4-x_t)-3+2x_t(x_t-2)\ln x_t)}{(x_t-1)^3}}{\cos^2\theta_W} \\
& -\frac{4(x(4+2x_t-\frac{x_t y}{z_H})-2)}{\Theta_8} - \frac{4(x_t(\frac{y(2-x)}{z_H}+3+\Theta_7)-4x) + 2\Theta_7 x_t\ln\Theta_7)}{x_t\Theta_7} \\
& +\frac{2(2-12x_t+21xt^2-12xt^3+x_t^4+(2-4x_t-2x_t^2+6x_t^3)\ln x_t)}{(x_t-1)^3} - 4x_t(1+2\ln\left[\Theta_8\right])\Bigg),
\end{aligned}$$

$$
\begin{aligned}
C^{h_0}_{Q_1}((\bar{\xi}^U_{N,tt})^3) &= -\frac{\bar{\xi}^D_{N,\tau\tau}(\bar{\xi}^U_{N,tt})^3 m_b y_t}{64\Theta_1(z_h) m^2_{h^0} m_t \pi^2(-1+y_t)^3}((-1+y_t)(\Theta_1(z_h)(y_t+1) \\
&+ 2(2x-1)(y_t-1)^2 z_h) - 2\Theta_1(z_h) y_t \ln y_t), \\
C^{h_0}_{Q_1}((\bar{\xi}^U_{N,tt})^2) &= -\frac{1}{32 m^2_{h^0}\pi^2}\bar{\xi}^D_{N,\tau\tau}\bar{\xi}^D_{N,bb}(\bar{\xi}^U_{N,tt})^2\Big(-\frac{\Theta_1(z_h)+y_t(xy-z_h)}{\Theta_1(z_h)} + \frac{(1-y_t+(-1+2y_t)\ln y_t}{-1+y_t} \\
&- 2\ln\Big[\frac{\Theta_1(z_h)}{z_h}\Big]\Big),
\end{aligned}
$$

$$
\begin{aligned}
C^{h^0}_{Q_1}(\bar{\xi}^U_{N,tt}) = -\frac{g^2\bar{\xi}^D_{N,\tau\tau}\bar{\xi}^U_{N,tt} x_t}{128\pi^2 m^2_{h^0} m_t}\Bigg(&\frac{x_t(8-9y_t)-x_t^3(y_t-2)+y_t(5y_t-4)+x_t^2(-4+2y_t+y_t^2)}{\Theta_5(x_t-1)} \\
-\frac{y_t x_t(2-x_t-y_t)}{z_h\Theta_5} &- \frac{4z_h(-1+x(2+x_t))-2xyx_t}{\Theta_2(z_h)} + \frac{2z_h(-2y_t+x(x_t+y_t))}{\Theta_3(z_h)} \\
+\frac{2z_h(x_t(x-1)+y_t(x+1))}{-\Theta_3(z_h)+(x-y)(x_t-y_t)z_h} &+ \frac{4(-1+x)xx_t y_t^2 z_h}{(\Theta_4(z_h)x_t-x(x_t-y_t)z_h)(\Theta_4(z_h)x_t-y(x_t-y_t)z_h)} \\
+\Big(4+\frac{(y_t-1)^2(x_t^3(3-10y_t)+7y_t^2-7x_t y_t(2+y_t)+3x_t^2(1+4y_t+2y_t^2))}{(x_t-1)} \\
+\frac{y_t x_t(-1+y_t)^2(-\Theta_6+4(x_t-y_t)y_t)}{z_h}\Big)\frac{\ln x_t}{\Theta_5^2} \\
-\frac{(x_t-1)^2(-10x_t y_t(y_t-1)+x_t^2(2y_t-1)+y_t^2(4y_t-5)-\frac{x_t y_t}{z_h}\Theta_6)\ln y_t}{\Theta_5^2} &- 4\ln\Big[\frac{\Theta_2(z_h)}{z_h}\Big]\Bigg),
\end{aligned}
$$

$$
\begin{aligned}
C^{h^0}_{Q_1}(\bar{\xi}^D_{N,bb}) = -\frac{g^2\bar{\xi}^D_{N,\tau\tau}\bar{\xi}^D_{N,bb} z_h}{64\pi^2 m^2_W x_t}\Bigg(&\frac{(x_t^2(\frac{y_t}{z_h}+1)+y_t-2x_t(y_t+1))\ln x_t}{(x_t-1)(x_t-y_t)} \\
+\frac{(x_t^2(\frac{y_t}{z_h}-1)+2y_t(y_t-1)+x_t(1-2y_t))\ln y_t}{(y_t-1)(x_t-y_t)} &- 2\ln\Big[\frac{\Theta_3(z_h)}{z_h}\Big] \\
+x_t\frac{y_t(x_t+2y(x-1))-z_h(x_t-2y_t(y-1)+2y\frac{y_t}{x_t})+x(yy_t+2z_h(y_t-1))}{\Theta_3(z_h)}&\Bigg), \qquad (19)
\end{aligned}
$$

where

$$
\begin{aligned}
\Theta_1(\omega) &= ((1-y+yy_t)\omega - x(yy_t+\omega(1-y_t)) \\
\Theta_2(\omega) &= \Theta_1(\omega, y_t\to x_t) \\
\Theta_3(\omega) &= (x_t(1-y)+y)y_t\omega - xx_t(yy_t+\omega(-1+y_t)) \\
\Theta_4(\omega) &= (y(1-y_t)+y_t)\omega - x(yy_t+\omega(-1+y_t)) \\
\Theta_5 &= (-1+x_t)(x_t-y_t)(-1+y_t) \\
\Theta_6 &= (-1+y)(y(-1+y_t)y_t) \\
\Theta_7 &= \frac{(x_t+y(1-x_t))z_h + x(z_h-x_t(y+z_h))}{x_t z_h} \\
\Theta_8 &= \frac{(1-y(1-x_t))z_h - x(x_t(y-z_h)+z_h)}{x_t z_h} \qquad (20)
\end{aligned}
$$

and

$$x_t = \frac{m_t^2}{m_W^2} \quad , \quad y_t = \frac{m_t^2}{m_{H^\pm}} \quad , \quad z_H = \frac{m_t^2}{m_{H^0}^2} \quad , \quad z_h = \frac{m_t^2}{m_{h^0}^2} \quad , \quad z_A = \frac{m_t^2}{m_{A^0}^2} \quad , \tag{21}$$

The explicit forms of the functions $F_{1(2)}(y_t)$, $G_{1(2)}(y_t)$, $H_1(y_t)$ and $L_1(y_t)$ in eq.(17) are given as

$$\begin{aligned}
F_1(y_t) &= \frac{y_t(7-5y_t-8y_t^2)}{72(y_t-1)^3} + \frac{y_t^2(3y_t-2)}{12(y_t-1)^4} \ln y_t \;, \\
F_2(y_t) &= \frac{y_t(5y_t-3)}{12(y_t-1)^2} + \frac{y_t(-3y_t+2)}{6(y_t-1)^3} \ln y_t \;, \\
G_1(y_t) &= \frac{y_t(-y_t^2+5y_t+2)}{24(y_t-1)^3} + \frac{-y_t^2}{4(y_t-1)^4} \ln y_t \;, \\
G_2(y_t) &= \frac{y_t(y_t-3)}{4(y_t-1)^2} + \frac{y_t}{2(y_t-1)^3} \ln y_t \;, \\
H_1(y_t) &= \frac{1-4sin^2\theta_W}{sin^2\theta_W}\,\frac{xy_t}{8}\left[\frac{1}{y_t-1} - \frac{1}{(y_t-1)^2}\ln y_t\right] \\
&- y_t\left[\frac{47y_t^2-79y_t+38}{108(y_t-1)^3} - \frac{3y_t^3-6y_t+4}{18(y_t-1)^4}\ln y_t\right] \;, \\
L_1(y_t) &= \frac{1}{sin^2\theta_W}\,\frac{xy_t}{8}\left[-\frac{1}{y_t-1} + \frac{1}{(y_t-1)^2}\ln y_t\right] \;.
\end{aligned} \tag{22}$$

Finally, the initial values of the coefficients in the model III are

$$\begin{aligned}
C_i^{2HDM}(m_W) &= C_i^{SM}(m_W) + C_i^H(m_W), \\
C_{Q_1}^{2HDM}(m_W) &= \int_0^1 dx \int_0^{1-x} dy\, (C_{Q_1}^{H^0}((\bar{\xi}^U_{N,tt})^2) + C_{Q_1}^{H^0}(\bar{\xi}^U_{N,tt}) + C_{Q_1}^{H^0}(g^4) + C_{Q_1}^{h^0}((\bar{\xi}^U_{N,tt})^3) \\
&+ C_{Q_1}^{h^0}((\bar{\xi}^U_{N,tt})^2) + C_{Q_1}^{h^0}(\bar{\xi}^U_{N,tt}) + C_{Q_1}^{h^0}(\bar{\xi}^D_{N,bb})), \\
C_{Q_2}^{2HDM}(m_W) &= \int_0^1 dx \int_0^{1-x} dy\, (C_{Q_2}^{A^0}((\bar{\xi}^U_{N,tt})^3) + C_{Q_2}^{A^0}((\bar{\xi}^U_{N,tt})^2) + C_{Q_2}^{A^0}(\bar{\xi}^U_{N,tt}) + C_{Q_2}^{A^0}(\bar{\xi}^D_{N,bb})) \\
C_{Q_3}^{2HDM}(m_W) &= \frac{m_b}{m_\tau \sin^2\theta_W}(C_{Q_1}^{2HDM}(m_W) + C_{Q_2}^{2HDM}(m_W)) \\
C_{Q_4}^{2HDM}(m_W) &= \frac{m_b}{m_\tau \sin^2\theta_W}(C_{Q_1}^{2HDM}(m_W) - C_{Q_2}^{2HDM}(m_W)) \\
C_{Q_i}^{2HDM}(m_W) &= 0 \;,\; i = 5, ..., 10.
\end{aligned} \tag{23}$$

Here, we present C_{Q_1} and C_{Q_2} in terms of the Feynmann parameters x and y since the integrated results are extremely large. Using these initial values, we can calculate the coefficients $C_i^{2HDM}(\mu)$ and $C_{Q_i}^{2HDM}(\mu)$ at any lower scale in the effective theory with five quarks, namely u, c, d, s, b similar to the SM case [13, 18, 19, 20]. For completeness, in the following we give the explicit expressions for $C_7^{eff}(\mu)$ and $C_9^{eff}(\mu)$.

$$C_7^{eff}(\mu) = C_7^{2HDM}(\mu) + Q_d\,(C_5^{2HDM}(\mu) + N_c\,C_6^{2HDM}(\mu)) \;,$$

$$+ \quad Q_u \, (\frac{m_c}{m_b} \, C_{12}^{2HDM}(\mu) + N_c \, \frac{m_c}{m_b} \, C_{11}^{2HDM}(\mu)) \ , \tag{24}$$

where the LO QCD corrected Wilson coefficient $C_7^{LO,2HDM}(\mu)$ is given by

$$\begin{aligned} C_7^{LO,2HDM}(\mu) &= \eta^{16/23} C_7^{2HDM}(m_W) + (8/3)(\eta^{14/23} - \eta^{16/23}) C_8^{2HDM}(m_W) \\ &+ C_2^{2HDM}(m_W) \sum_{i=1}^{8} h_i \eta^{a_i} \ , \end{aligned} \tag{25}$$

and $\eta = \alpha_s(m_W)/\alpha_s(\mu)$, h_i and a_i are the numbers which appear during the evaluation [19].

The Wilson coefficient $C_9^{eff}(\mu)$ is :

$$\begin{aligned} C_9^{eff}(\mu) &= C_9^{2HDM}(\mu) \\ &+ h(z,s) \left(3C_1(\mu) + C_2(\mu) + 3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu)\right) \\ &- \frac{1}{2} h(1,s) \left(4C_3(\mu) + 4C_4(\mu) + 3C_5(\mu) + C_6(\mu)\right) \\ &- \frac{1}{2} h(0,s) \left(C_3(\mu) + 3C_4(\mu)\right) + \frac{2}{9} \left(3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu)\right) \ . \end{aligned} \tag{26}$$

Here the functions $h(u,s)$ are given by

$$\begin{aligned} h(u,s) &= -\frac{8}{9} \ln \frac{m_b}{\mu} - \frac{8}{9} \ln u + \frac{8}{27} + \frac{4}{9} x \\ & \quad -\frac{2}{9} (2+x) |1-x|^{1/2} \begin{cases} \left(\ln \left| \frac{\sqrt{1-x}+1}{\sqrt{1-x}-1} \right| - i\pi \right), & \text{for } x \equiv \frac{4u^2}{s} < 1 \\ 2 \arctan \frac{1}{\sqrt{x-1}}, & \text{for } x \equiv \frac{4u^2}{s} > 1, \end{cases} \end{aligned} \tag{27}$$

$$h(0,s) = \frac{8}{27} - \frac{8}{9} \ln \frac{m_b}{\mu} - \frac{4}{9} \ln s + \frac{4}{9} i\pi \ , \tag{28}$$

with $u = \frac{m_c}{m_b}$.

Finally, the Wilson coefficient $C_{10}(\mu)$ is the same as $C_{10}(m_W)$ and $C_{Q_1}(\mu)$, $C_{Q_2}(\mu)$ are given by [13]

$$C_{Q_i}(\mu) = \eta^{-12/23} \, C_{Q_i}(m_W) \ , \ i = 1, 2 \ . \tag{29}$$

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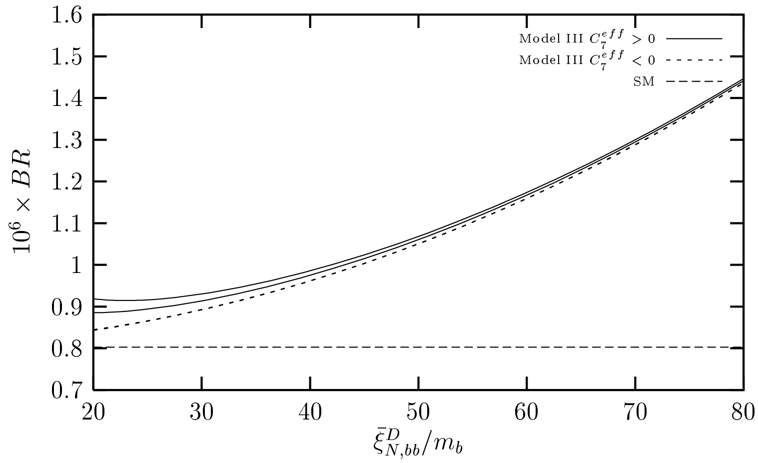

Figure 1: BR as a function of $\bar{\xi}^D_{N,bb}/m_b$ for $\bar{\xi}^D_{N,\tau\tau} = 200\, GeV$ in case of the ratio $|r_{tb}| < 1$.

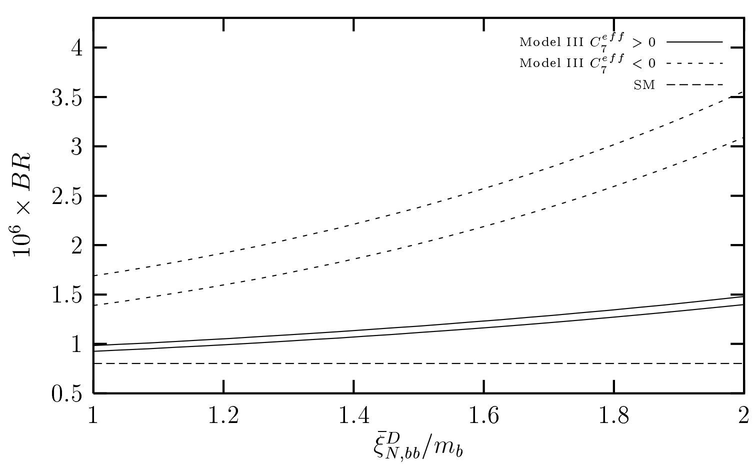

Figure 2: BR as a function of $\bar{\xi}^D_{N,bb}/m_b$ for $\bar{\xi}^D_{N,\tau\tau} = 20\, GeV$ in case of the ratio $r_{tb} > 1$.

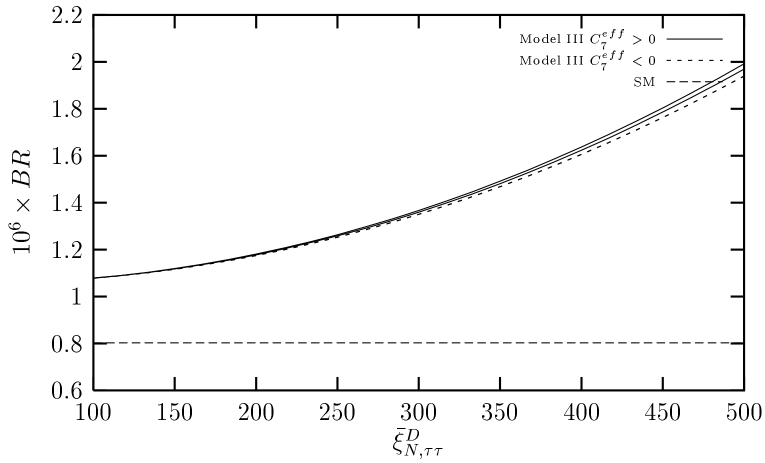

Figure 3: BR as a function of $\bar{\xi}^{D}_{N,\tau\tau}$, for $\bar{\xi}^{D}_{N,bb} = 40\, m_b$ in case of the ratio $|r_{tb}| < 1$.

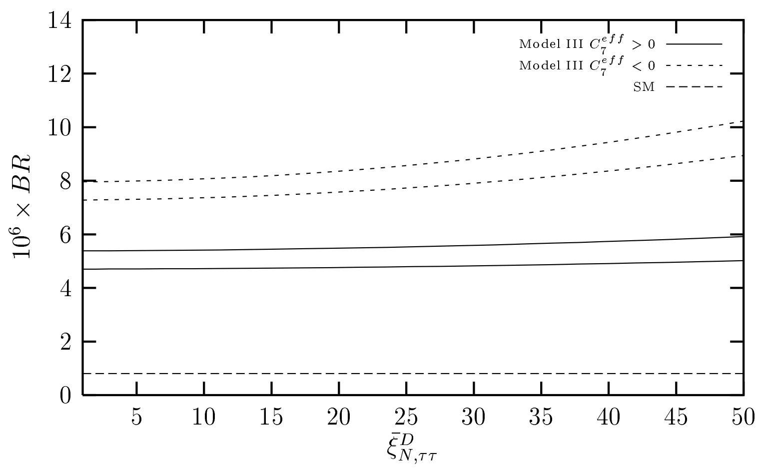

Figure 4: BR as a function of $\bar{\xi}^{D}_{N,\tau\tau}$, for $\bar{\xi}^{D}_{N,bb} = 3\, m_b$ in case of the ratio $r_{tb} > 1$.